The Transverse Energy as a Barometer of a Saturated Plasma

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The evolution of the gluon plasma produced with saturation initial conditions is calculated via Boltzmann transport theory for nuclear collisions at high energy. The saturation scale increases with A and $\sqrt{s}$, and thus we find that the perturbative rescattering rate decreases relative to the initial longitudinal expansion rate of the plasma. The effective longitudinal pressure remains significantly below the lattice QCD pressure until the plasma cools to near the confinement scale. Therefore, the transverse energy per unit of rapidity and its dependence on beam energy provides a sensitive test of gluon saturation models: the fractional transverse energy loss due to final state interactions is smaller and exhibits a weaker energy dependence than if ideal (nondissipative) hydrodynamics applied throughout the evolution.

In collisions of heavy ions at high energy a large number of gluons is liberated from the nuclear wave functions. The "plasma" is produced from copious minijet gluons at central rapidity, $y \simeq 0$, with transverse momentum $p_T > p_0$. For large p_0, the produced gluon plasma is dilute. As p_0 decreases, however, the density of gluons increases rapidly due to the increase of $G(x, p_T^2)$ as $x \approx 2p_T/\sqrt{s}$ decreases. It has been conjectured [1,2] that below some transverse momentum scale $p_0 \leq p_{\rm sat}$ the phase-space density of produced gluons may saturate since $gg \to g$ recombination could limit further growth of the structure functions. Phenomenologically, this condition may arise when gluons (per unit rapidity and transverse area) become closely packed and fill the available nuclear interaction transverse area. The saturation scale $p_{\rm sat}$ can thus be estimated from $\mathrm{d}N(p_{\rm sat})/\mathrm{d}y = p_{\rm sat}^2 R_A^2/\beta$, where $\beta \sim 1$. For $\beta = 1$ the solution reported in EKRT [3] was $p_{\rm sat} \approx 0.208 A^{\,0.128} \sqrt{s}^{\,0.191}$, where $p_{\rm sat}$ and $\sqrt{s}$ are in units of GeV, and $C_1 \equiv \mathrm{d}E_T/\mathrm{d}y/p_{\rm sat}\mathrm{d}N/\mathrm{d}y \approx 1.34 A^{-0.007} \sqrt{s}^{0.021}$. Our focus is to investigate whether the final observed $\mathrm{d}E_T^f/\mathrm{d}y$ can be used to test the predicted A and $\sqrt{s}$ dependence of the initial $\mathrm{d}E_T^i/\mathrm{d}y$.

Different gluon saturation models based on classical Yang-Mills equations [2,4] suggest that the factor β may vary parametrically as $\beta(p_{\rm sat}) = 4\pi\alpha(p_{\rm sat})N_c/c(N_c^2-1)$, where $c \sim 1$ is a nonperturbative factor proportional to the fraction of the initial gluons in the nucleus which are liberated. This factor was recently estimated [5] to be $c \approx 1.2 - 1.5$. On the other hand, PHOBOS and PHENIX data [6] require $c \approx 1.9$ if one assumes entropy conserving hadronization, and for α running with $p_{\rm sat}$; that is, $\sim 50\%$ additional entropy must be produced during nonideal expansion of the saturated gluon plasma. In the approach of [7], the ratio of final to initial multiplicity (or entropy) also grows with energy as $\sim \alpha^{-2/5}$. This, in turn, suggests that less mechanical work is being performed by the longitudinal expansion than predicted by ideal fluid dynamics [8], and so the E_t in the final state should be closer to it's initial value.

If the expansion proceeds in approximate local equilibrium with pressure $p = c^2\epsilon$ and speed of sound c, then the energy density, $\epsilon(\tau)$, must decrease faster than the expansion rate $\Gamma_{exp} = 1/\tau$ and leads to a bulk transverse energy loss

$$\frac{E_T(\tau)}{E_T(\tau_0)} = \frac{\tau\epsilon}{\tau_0\epsilon_0} = \left(\frac{\tau_0}{\tau}\right)^{\delta} \quad . \tag{1}$$

If local equilibrium is maintained during the evolution $\delta = c^2$. In contrast, if the system expands too rapidly to maintain local equilibrium, then the effective pressure is reduced due to dissipation. The extreme asymptotically free plasma case corresponds to free streaming with $\delta = 0$. E_T thus provides an important barometric observable that probes the (longitudinal) pressure in the plasma.

The relevant relaxation rate is given by the fractional energy loss per unit length, $\Gamma_{rel} = \mathrm{d}\log E/\mathrm{d}z$, which receives a contribution both from elastic and inelastic scattering. The relaxation rate is approximately given by [8]

$$\Gamma_{rel} \approx 9\pi\alpha^2\frac{\rho^3}{\epsilon^2}\left(\log\frac{1}{\alpha} + \frac{27}{2\pi^2}\right) \equiv K_{in}9\pi\alpha^2\frac{\rho^3}{\epsilon^2}\log\frac{1}{\alpha} \ , \tag{2}$$

where we lumped energy loss from radiation into K_{in}, the inelasticity K-factor. $K_{in} = 1$ corresponds to purely elastic scattering, while $K_{in} = 2$, for example, corresponds to twice the lowest-order elastic scattering rate.

At the initial time $\tau_0 = 1/p_{\rm sat}$, $\overline{s}/2 = \epsilon_0^2/\rho_0^2 = C_1^2 p_{\rm sat}^2$. Therefore, noting that the comoving gluon density at time τ_0 is $\rho_0 = p_{\rm sat}^3/\pi\beta$, the ratio of the relaxation rate to the expansion rate is given by

$$\frac{\Gamma_{rel}}{\Gamma_{exp}} = K_{in}\frac{9\alpha^2}{\beta C_1^2}\log\frac{1}{\alpha} \quad . \tag{3}$$

While $\Gamma_{rel} \propto p_{\rm sat}$ increases as a power of the energy, the Bjorken boundary conditions force the system to expand londitudinally initially also at an increasing rate $\Gamma_{exp}(\tau_0) = p_{\rm sat}$. The essential quantity that fixes the magnitude of the effective pressure relative to that predicted by lattice QCD (LQCD) is the ratio of rates in Eq. (3), which dimensionally is simply a function of $\alpha(p_{\rm sat})$. The asymptotic freedom property of QCD therefore requires that this ratio vanishes as $\sqrt{s} \to \infty$. In (3) the rate of how fast it vanishes is controlled by $K_{in}\alpha^2(p_{\rm sat})/\beta(p_{\rm sat})$. Therefore, with saturation initial conditions, asymptotic freedom reduces the *effective* pressure acting at early times $\tau \sim \tau_0$ and causes the initial evolution to deviate from ideal hydrodynamics for a time interval that increases with energy [8].

For a more quantitative estimate of the transverse energy loss due to longitudinal work, we employ the Boltzmann equation in relaxation time approximation [9]. As initial condition, we assume that the initially produced partons have a vanishing longitudinal momentum spread in the comoving frame, i.e. we assume a strong correlation between space-time rapidity and momentum space rapidity; the accuracy of this approximation at the finite RHIC energy remains to be checked within more elaborate models. In any case, our gluon plasma thus starts at zero longitudinal pressure, and we shall follow its evolution up to the point where it reaches the p/ϵ-ratio from LQCD [8]. In Fig. 1 we show the effective longitudinal pressure as a function of the energy density for $\sqrt{s} = 20, 200, 5400A$ GeV saturation initial conditions [8]. Initially p/ϵ starts at zero and remains

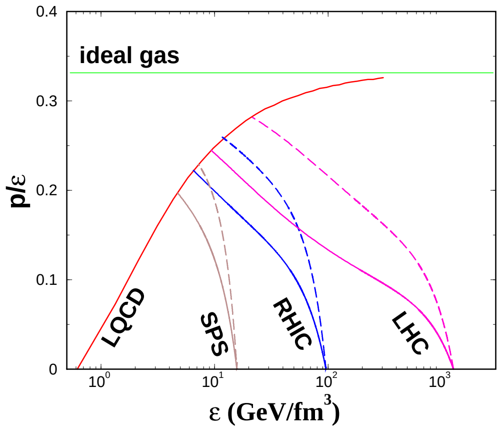

Figure 1. The ratio of the effective longitudinal pressure to the energy density along the dynamical path is shown for SPS, RHIC, and LHC saturation initial conditions[3]. Solid (dashed) curves are for $\beta = 1$ and $K_{in} = 1(2)$. The LQCD equation of state [10] is also shown for comparison.

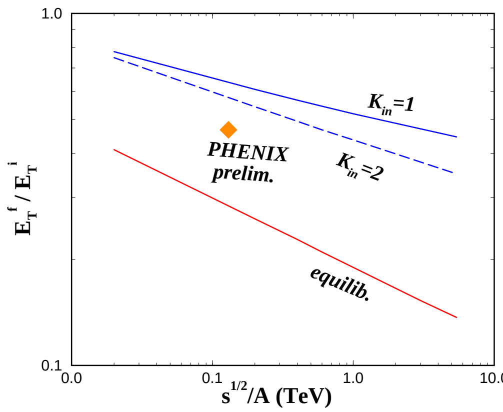

Figure 2. The ratios of the final to the initial transverse energy per unit rapidity are shown as a function of beam energy for central Au+Au collisions. The initial value corresponds to the EKRT parametrization ($\beta = 1$) and the transport results are for $K_{in} = 1, 2$. The final transverse energy for an ideal gas of gluons in equilibrium (throughout the evolution) is also shown.

small for a large time relative to $\tau_0 = 1/p_{\rm sat}$ because the plasma is torn apart by the initial rapid longitudinal expansion. The effective pressure approaches the LQCD curve from below and reaches it at a time $\tau_L \approx 1-2$ fm at RHIC, $\sqrt{s} = 200A$ GeV, by which time the energy density has dropped by an order of magnitude, $\epsilon_L \equiv \epsilon(\tau_L) = 6.5-12$ GeV/fm^3. For LHC $\sqrt{s} = 5400A$ GeV, $\tau_L = 3-7$ fm during which the energy density falls by almost two orders of magnitude to $\epsilon_L = 9.5-21.5$ GeV/fm^3. The quoted intervals correspond to $K_{in} = 1-2$ using the EKRT parametrization [3]. The contrast between the dynamical path followed by the saturated plasma compared to the equilibrium equation of state is striking. A qualitatively similar behavior of the early logitudinal pressure has also been found from solutions of diffusion equations [11].

The main experimentally observable consequence of the reduced effective pressure is shown in the right panel. The ratio $E_T^f/E_T^i = \tau_f\epsilon_f/\tau_0\epsilon_0$ has been obtained from the solution of the transport equation assuming free streaming at $\epsilon_f = 2$ GeV/fm^3 which corresponds roughly to $T \simeq T_c$. τ_f is determined assuming hydrodynamic expansion from the point were the LQCD pressure curve is reached. On the other hand, for an ideal gas of gluons produced at time τ_0, the final observed transverse energy for 1+1 dimensional adiabatic expansion would be

$$\frac{E_T^f}{E_T^i} = \frac{\tau_f\epsilon_f}{\tau_0\epsilon_0} = \frac{\tau_f(T_f s_f - p_f)}{\tau_0(T_0 s_0 - p_0)} = \frac{T_f}{T_0}. \qquad (4)$$

Clearly, for $T_f \sim T_c$ one would observe a much smaller transverse energy in the final state than in the initial state. Moreover, E_T^f/E_T^i would also have significantly stronger energy

dependence such that E_T^f deviates more and more from E_T^i with increasing $\sqrt{s}$. In this sense isentropic hydrodynamics erases information on the interesting initial conditions via this observable. The solutions of the transport equations clearly show a smaller decrease of E_T^f and of the logarithmic slope, $\kappa = \mathrm{d}\log E_T^f/\mathrm{d}\log\sqrt{s}$, due to final state interactions. We find that $\kappa = 0.50$ for the evolution with $K_{in} = 1$, $\kappa = 0.46$ with $K_{in} = 2$, while $\kappa = 0.40$ with isentropic expansion, eq. (4). For comparison, the initial EKRT saturated $E_T^i = \pi R_A^2 \tau_0 \epsilon_0$ scales with the higher power $\kappa = 0.59$. The fractional transverse energy loss is thus less dependent on energy than for entropy conserving expansion for which $E_T^f/E_T^i \propto 1/T_0 \propto 1/\sqrt{s}^{\,0.2}$. This is due to the increasingly long time spent far from equilibrium in Fig. 1 as the beam energy increases.

The results in Fig. 2 are encouraging from the point of view of searching for evidence of gluon saturation in nuclei at high energies. Experimental data on $\mathrm{d}E_T/\mathrm{d}y$ or $\mathrm{d}E_T/\mathrm{d}\eta$ for central Au+Au collisions at RHIC will soon provide a new test of saturation and non-saturation models at those energies. Since we predict that dissipative effects reduce considerably the effective longitudinal pressure in Fig. 1, the beam-energy dependence of the transverse energy is expected to reflect much more accurately the predicted power law dependence of the initial conditions as seen in Fig. 2. We have also shown the preliminary result from PHENIX [12], $\mathrm{d}E_t/\mathrm{d}\eta \approx 570$ GeV for the 2% most central Au+Au events at $\sqrt{s} = 130A$ GeV, scaled to $\mathrm{d}E_t/\mathrm{d}y$ and divided by the initial E_t from EKRT. It would be most interesting to have data both at higher and lower energy to determine the slope κ, and compare to the expectation for a saturated plasma. The energy and A systematics of the bulk calorimetric observable, $\mathrm{d}E_T/\mathrm{d}y$, will be a sensitive test of saturation models of gluon plasmas produced in the RHIC to LHC energy range.

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